Final Exam – 2010-2011

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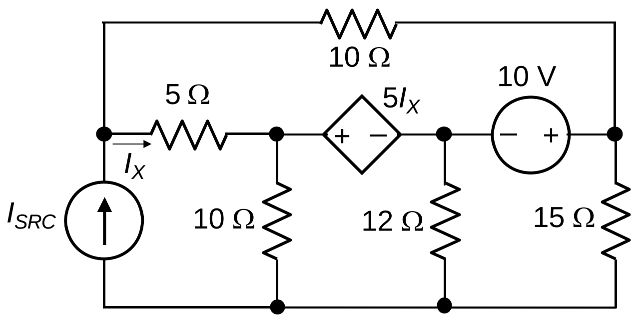
1. Determine I_X, assuming I_{SRC} = 1 A (Hint: write one KVL equation and one KCL equation).
   A. 4 A
   B. 2 A
   C. 3 A
   D. 1 A
   E. 5 A



Solution: Let the current through the 10 Ω resistor be I_Y. From KVL around the mesh abcda: $5I_X + 5I_X - 10 - 10I_Y = 0$, which gives, $I_Y = I_X - 1$. From KCL at node a, $I_{SRC} = I_Y + I_X$, or $I_{SRC} = 2I_X - 1$. It follows that $I_X = (I_{SRC} + 1)/2$.

Version 1: I_{SRC} = 1 A, I_X = (1 + 1)/2 = 1 A

Version 2: I_{SRC} = 1.5 A, I_X = (1.5 + 1)/2 = 1.25 A

Version 3: I_{SRC} = 2 A, I_X = (2 + 1)/2 = 1.5 A

Version 4: I_{SRC} = 2.5 A, I_X = (2.5 + 1)/2 = 1.75 A

Version 5: I_{SRC} = 3 A, I_X = (3 + 1)/2 = 2 A

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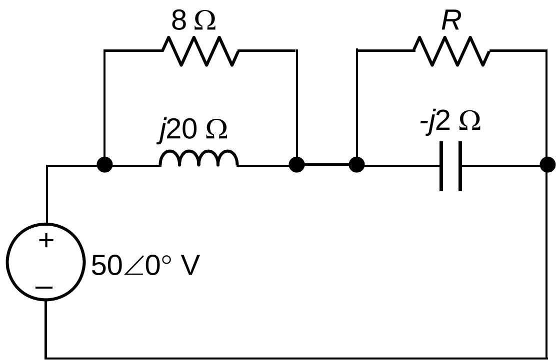
2. In the circuit shown, the capacitance absorbs -200 VAR. Determine the average power dissipated in *R* if *R* = 5 Ω.
   A. 57.1 W
   B. 80 W
   C. 44.4 W
   D. 66.7 W
   E. 50 W



Solution: $Q = -BV_{\text{rms}}^2$, where V_{rms} is the rms voltage across *R* and *C,* and *B* = -1/*X* = 1/2 S.

Substituting, -200 = $-\frac{1}{2}V_{\text{rms}}^2$, and V_{rms} = 20 V. It follows that $P_R = \frac{V_{\text{rms}}^2}{R}$.

Version 1: *R* = 5 Ω, $P_R = \frac{400}{5} = 80$ W

Version 2: *R* = 6 Ω, $P_R = \frac{400}{6} = 66.7$ W

Version 3: R = 7 Ω, P_R = $\frac{400}{7} = 57.1$ W

Version 4: R = 8 Ω, P_R = $\frac{400}{8} = 50$ W

Version 5: R = 9 Ω, P_R = $\frac{400}{9} = 44.4$ W

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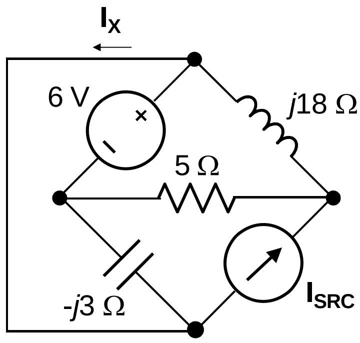
3. Determine $\mathbf{I_X}$ assuming $\mathbf{I_{SRC}}$ = j A.
   A. j6 A
   B. -j3 A
   C. j3 A
   D. -j6 A
   E. j4 A



Solution: The voltage across the -j3 Ω capacitor is 6 V and the current through this capacitor, directed upwards is j2 A. It follows that $\mathbf{I_X} = \mathbf{I_{SRC}} + j2$ A.

Version 1: $\mathbf{I_{SRC}} = j$ A, $\mathbf{I_X} = j + j2 = j3$ A

Version 2: $\mathbf{I_{SRC}} = j2$ A, $\mathbf{I_X} = j2 + j2 = j4$ A

Version 3: $\mathbf{I_{SRC}} = j3$ A, $\mathbf{I_X} = j3 + j2 = j5$ A

Version 4: $\mathbf{I_{SRC}} = j4$ A, $\mathbf{I_X} = j4 + j2 = j6$ A

Version 5: $\mathbf{I_{SRC}} = j5$ A, $\mathbf{I_X} = j5 + j2 = j7$ A

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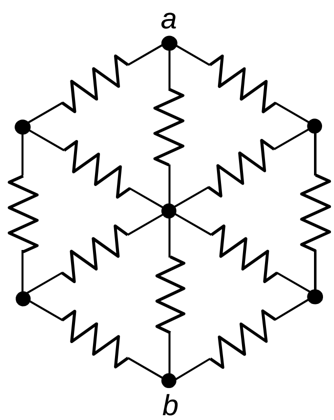
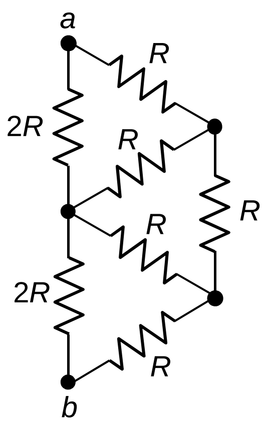
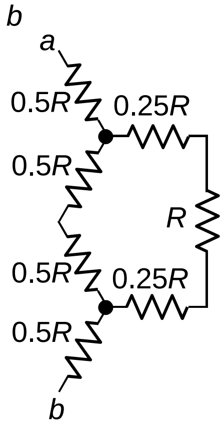
4. Determine the resistance between nodes a and b assuming that all resistances are 1 Ω.
   A. 4 Ω
   B. 2.4 Ω
   C. 3.2 Ω
   D. 1.6 Ω
   E. 0.8 Ω



Solution: The resistances can be split into two halves in parallel, where the resistances in the middle become 2R. Applying Δ-Y transformation,





the resistances become as shown. It follows that $2R_{ab} = 0.5R + R||1.5R + 0.5R = R + 0.6R = 1.6R$, so that $R_{ab} = 0.8R$.

Version 1: $R = 1\ \Omega$, $R_{ab} = 0.8\times1 = 0.8\ \Omega$

Version 2: $R = 2\ \Omega$, $R_{ab} = 0.8\times2 = 1.6\ \Omega$

Version 3: $R = 3\ \Omega$, $R_{ab} = 0.8\times3 = 2.4\ \Omega$

Version 4: $R = 4\ \Omega$, $R_{ab} = 0.8\times4 = 3.2\ \Omega$

Version 5: $R = 5\ \Omega$, $R_{ab} = 0.8\times5 = 4\ \Omega$

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5. Two coils are tightly coupled to a high-permeability core, so that the leakage flux is negligibly small. If coil 1 has 100 turns and an inductance of 10 mH, and the mutual inductance is 12.5 mH, determine the number of turns of coil 2.
   A. 125
   B. 250
   C. 150
   D. 175
   E. 200

Solution: From the definitions of self and mutual inductance, with negligible leakage flux, $L_1 = \frac{N_1\phi_{21}}{i_1}$ and $M = \frac{N_2\phi_{21}}{i_1}$. It follows that $N_2 = \frac{M}{L_1}N_1 = 10M$.

Version 1: $M = 12.5$ mH, $N_2 = 10\times12.5 = 125$ turns

Version 2: $M = 15$ mH, $N_2 = 10\times15 = 150$ turns

Version 3: $M = 17.5$ mH, $N_2 = 10\times17.5 = 175$ turns

Version 4: $M = 20$ mH, $N_2 = 10\times20 = 200$ turns

Version 5: $M = 22.5$ mH, $N_2 = 10\times22.5 = 225$ turns

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6. Determine the inductance of coil 2 of the preceding problem.
   A. 22.5 mH
   B. 30.63 mH
   C. 15.63 mH
   D. 40 mH
   E. 50.63 mH

Solution: Since the coils are tightly coupled to the core, k = 1, so that $M^2 = L_1L_2$, or

$L_2 = \frac{M^2}{L_1} = 0.1M^2$ mH. It also follows from the solution of the preceding problem that

$N_1 = \frac{M}{L_2}N_2$. Dividing, $L_2 = L_1\left(\frac{N_2}{N_1}\right)^2 = 0.1M^2$

Version 1: M = 12.5 mH, $L_2 = 0.1\times(12.5)^2$ = 15.625 mH

Version 2: M = 15 mH, $L_2 = 0.1\times(15)^2$ = 22.5 mH

Version 3: M = 17.5 mH, $L_2 = 0.1\times(17.5)^2$ = 30.625 mH

Version 4: M = 20 mH, $L_2 = 0.1\times(20)^2$ = 40 mH

Version 5: M = 22.5 mH, $L_2 = 0.1\times(22.5)^2$ = 50.625 mH

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7. A D'Arsonval movement has a resistance of R Ω and a full-scale deflection of 100 μA. Determine the shunt resistance that will result in a full-scale deflection of 150 μA, assuming R = 50 Ω.
   A. 150 Ω
   B. 200 Ω
   C. 300 Ω
   D. 100 Ω
   E. 250 Ω

Solution: At full-scale deflection, the voltage drop across the movement and shunt is (R Ω)×(100 μA) = 100R μV. The shunt has to pass 50 μA, so its resistance is $R_{shunt} = 100R/50 = 2R$ Ω.

Version 1: R = 50 Ω, R_{shunt} = 2×50 = 100 Ω

Version 2: R = 75 Ω, R_{shunt} = 2×75 = 150 Ω

Version 3: R = 100 Ω, R_{shunt} = 2×100 = 200 Ω

Version 4: R = 125 Ω, R_{shunt} = 2×125 = 250 Ω

Version 5: R = 150 Ω, R_{shunt} = 2×150 = 300 Ω

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8. When a 9950 Ω resistance is connected in series with a D'Arsonval movement of unknown resistance and full-scale deflection current, a voltage of 1 V across the series combination gives a certain full-scale deflection. If an additional 10,000 Ω is connected in series with the combination, 2 V are required for full-scale deflection. Determine the resistance of the D'Arsonval movement.

A. 150 Ω

B. 100 Ω

C. 75 Ω

D. 125 Ω

E. 50 Ω

Solution: Let the resistance of the movement be R_m, its FSD current be I_{FSD}, and the FSD voltage with series resistance be V_{FSD}. Then $I_{FSD}(R + R_m) = V_{FSD}$, and $I_{FSD}(10{,}000 + R + R_m) = 2V_{FSD}$. It follows that $R + R_m = 10{,}000$, or $R_m = 10{,}000 – R$.

Version 1: R = 9950 Ω, R_m = 10,000 – 9950 = 50 Ω

Version 2: R = 9925 Ω, R_m = 10,000 – 9925 = 75 Ω

Version 3: R = 9900 Ω, R_m = 10,000 – 9900 = 100 Ω

Version 4: R = 9875 Ω, R_m = 10,000 – 9875 = 125 Ω

Version 5: R = 9850 Ω, R_m = 10,000 – 9850 = 150 Ω

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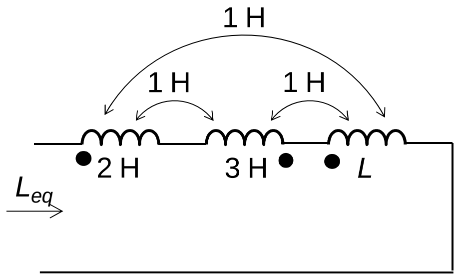
9. Determine L_{eq} if L = 1 H.



A. 6 H

B. 4 H

C. 8 H

D. 7 H

E. 5 H

Solution: Consider that a voltage **V** is applied, causing a current **I** to flow. $\mathbf{V} = j\omega\mathbf{I}[(2 – 1 + 1) + (3 – 1 – 1) + (L + 1 – 1)]$; $L_{eq} = 3 + L$.

Version 1: L = 1 H, L_{eq} = 4 H

Version 2: L = 2 H, L_{eq} = 5 H

Version 3: L = 3 H, L_{eq} = 6 H

Version 4: L = 4 H, L_{eq} = 7 H

Version 5: L = 5 H, L_{eq} = 8 H.

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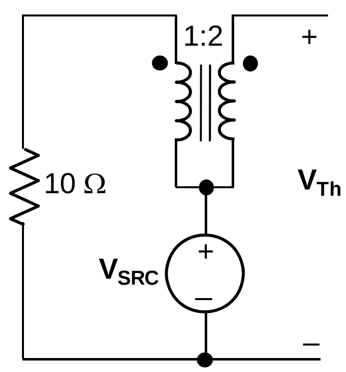
10. Determine $\mathbf{V_{Th}}$, assuming $\mathbf{V_{SRC}}$ = 1∠0° V



A. -1∠0° V

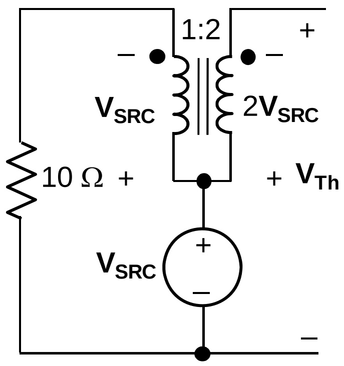
B. 1∠0° V

C. -2∠0° V

D. 2∠0° V

E. 4∠0° V

Solution: On open circuit, no current flows. The primary voltage is $\mathbf{V_{SRC}}$ as shown, and $\mathbf{V_{Th}}$ = -$\mathbf{V_{SRC}}$.



Version 1: $\mathbf{V_{SRC}} = 1\angle 0°$ V, $\mathbf{V_{Th}} = -1\angle 0°$ V

Version 2: $\mathbf{V_{SRC}} = 2\angle 0°$ V, $\mathbf{V_{Th}} = -2\angle 0°$ V

Version 3: $\mathbf{V_{SRC}} = 3\angle 0°$ V, $\mathbf{V_{Th}} = -3\angle 0°$ V

Version 4: $\mathbf{V_{SRC}} = 4\angle 0°$ V, $\mathbf{V_{Th}} = -4\angle 0°$ V

Version 5: $\mathbf{V_{SRC}} = 5\angle 0°$ V, $\mathbf{V_{Th}} = -5\angle 0°$ V

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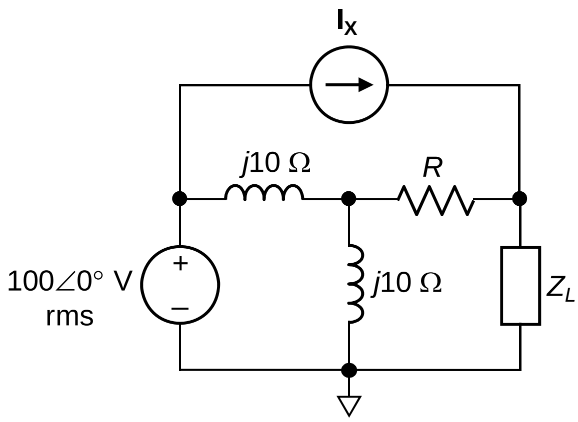
11. Determine Z_L for maximum average power delivered to it if $R = 5\ \Omega$ and $\mathbf{I_X} = k\angle -45°$ where $k = \sqrt{2}$ A rms.



   A. 10 + *j*10 Ω
   B. 5 + *j*5 Ω
   C. 5 – *j*5 Ω
   D. 10 – *j*10 Ω
   E. 15 – *j*15 Ω

Solution: Z_{Th} is $(R + j5)\ \Omega$. Hence, Z_L for maximum power transfer is $(R - j5)\ \Omega$.

Version 1: $R = 5\ \Omega$, $Z_{L\max} = (5 - j5)\ \Omega$

Version 2: $R = 6\ \Omega$, $Z_{L\max} = (6 - j5)\ \Omega$

Version 3: $R = 7\ \Omega$, $Z_{L\max} = (7 - j5)\ \Omega$

Version 4: $R = 8\ \Omega$, $Z_{L\max} = (8 - j5)\ \Omega$

Version 5: $R = 9\ \Omega$, $Z_{L\max} = (9 - j5)\ \Omega$

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12. Determine the maximum average power delivered to Z_L in Problem 11, assuming that $R = 5\ \Omega$ and $\mathbf{I_X}$ is as in Problem 11.
   A. 90 W
   B. 200 W
   C. 320 W
   D. 180 W
   E. 245 W

Solution: $\mathbf{V_{Th}}$ as seen by Z_L is determined from superposition as $\frac{j10}{j10+j10}\times 100\angle 0^\circ$ +

$(5+j10\,||\,j10)\mathbf{I_X} = 50\angle 0^\circ + 5(1+j)\mathbf{I_X} = 50\angle 0^\circ + (5\sqrt{2}\angle 45^\circ)\times k\angle -45^\circ = 50+5\sqrt{2}k$;

$$P_{L\max} = \frac{\left(50+5\sqrt{2}k\right)^2}{4R_{Th}} = \frac{\left(50+5\sqrt{2}k\right)^2}{20}$$

Version 1: $k=\sqrt{2}$ A, $P_{L\max} = \frac{\left(50+5\sqrt{2}\times\sqrt{2}\right)^2}{20} = \frac{(60)^2}{20}$ = 180 W

Version 2: $k=2\sqrt{2}$ A, $P_{L\max} = \frac{\left(50+5\sqrt{2}\times 2\sqrt{2}\right)^2}{20} = \frac{(70)^2}{20}$ = 245 W

Version 3: $k=3\sqrt{2}$ A, $P_{L\max} = \frac{\left(50+5\sqrt{2}\times 3\sqrt{2}\right)^2}{20} = \frac{(80)^2}{20}$ = 320 W

Version 4: $k=4\sqrt{2}$ A, $P_{L\max} = \frac{\left(50+5\sqrt{2}\times 4\sqrt{2}\right)^2}{20} = \frac{(90)^2}{20}$ = 405 W

Version 5: $k=5\sqrt{2}$ A, $P_{L\max} = \frac{\left(50+5\sqrt{2}\times 5\sqrt{2}\right)^2}{20} = \frac{(100)^2}{20}$ = 500 W

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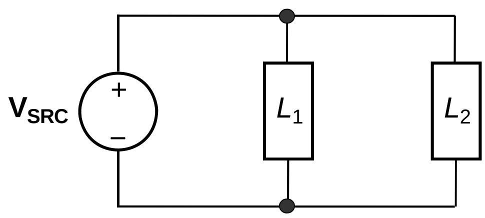
13. Load L_1 absorbs 15 kVA at 0.6 p.f. lagging, whereas Load L_2 absorbs 4.8 kW at 0.8 p.f. leading. If $\mathbf{V_{SRC}}$ = 200∠0° V rms at f = 50 Hz, determine the capacitor that must be connected in parallel with L_1 and L_2 to have maximum magnitude of current through the source.



A. 0.67 mF
B. 0.55 mF
C. 0.34 mF
D. 0.46 mF
E. 1.24 mF

Solution: The reactive power absorbed by L_1 is 15×0.8 kVAR = 12 kVAR, whereas the reactive power absorbed by L_2 is $-\frac{4.8}{0.8}\times 0.6 = -3.6$ kVAR. For maximum magnitude of source current, the p.f. should be unity. The capacitor must add a reactive power of -(12 – 3.6) = -8400 VAR. hence, -8400 = $-\omega C\times|\mathbf{V_{SRC}}|^2$, or $C = \frac{8400}{100\pi\,|\mathbf{V_{SRC}}|^2} = \frac{84}{\pi\,|\mathbf{V_{SRC}}|^2}$.

Version 1: $|V_{SRC}|$ = 200 V, $C = \dfrac{84}{\pi(200)^2} \equiv 0.67$ mF

Version 2: $|V_{SRC}|$ = 220 V, $C = \dfrac{84}{\pi(220)^2} \equiv 0.55$ mF

Version 3: $|V_{SRC}|$ = 240 V, $C = \dfrac{84}{\pi(240)^2} \equiv 0.46$ mF

Version 4: $|V_{SRC}|$ = 260 V, $C = \dfrac{84}{\pi(200)^2} \equiv 0.40$ mF

Version 5: $|V_{SRC}|$ = 280 V, $C = \dfrac{84}{\pi(200)^2} \equiv 0.34$ mF

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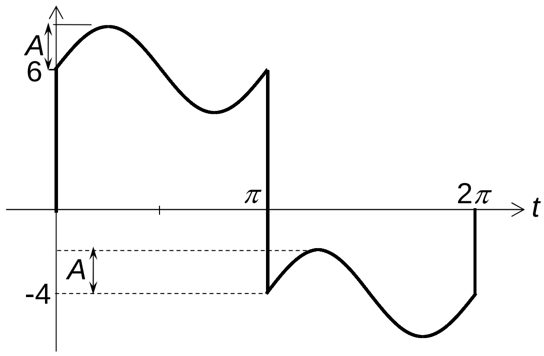
14. A periodic current is shown, where over a period,

$i = 6 + A\sin 2t \qquad 0 \le t \le \pi$

$i = -4 + A\sin 2(t - \pi) \qquad \pi 0 \le t \le 2\pi$

Determine the rms value of i if A = 1 A.



A. 5.83 A
B. 5.15 A
C. 6.20 A
D. 5.29 A
E. 5.52 A

Solution: The waveform consists of three components: i) a dc component of 1 A, ii) a square wave of 5 V amplitude, and iii) a sinusoidal wave of amplitude A. It follows that the rms value is $I = \sqrt{1^2 + 5^2 + A^2/2} = \sqrt{26 + A^2/2}$ A

Version 1: A = 1; $I = \sqrt{26 + A^2/2} = \sqrt{26.5} = 5.15$ A

Version 2: A = 2; $I = \sqrt{26 + A^2/2} = \sqrt{28} = 5.29$ A

Version 3: A = 3; $I = \sqrt{26 + A^2/2} = \sqrt{30.5} = 5.52$ A

Version 4: A = 4; $I = \sqrt{26 + A^2/2} = \sqrt{34} = 5.83$ A

Version 5: A = 5; $I = \sqrt{26 + A^2/2} = \sqrt{38.5} = 6.20$ A.

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15. The current waveform of the preceding problem is applied to a 2 Ω resistor in parallel with a very large capacitor. Determine the voltage across the parallel combination.

A. 2.5 V

B. 2 V

C. 3 V

D. 4 V

E. 3.5 V

Solution: The ac voltage will be negligibly small. The dc voltage is the dc component of current multiplied by *R*, or $V = 1\times R$.

Version 1: $R = 2\ \Omega$; $V = R = 2$ V

Version 2: $R = 2.5\ \Omega$; $V = R = 2.5$ V

Version 3: $R = 3\ \Omega$; $V = R = 3$ V

Version 4: $R = 3.5\ \Omega$; $V = R = 3.5$ V

Version 5: $R = 4\ \Omega$; $V = R = 4$ V

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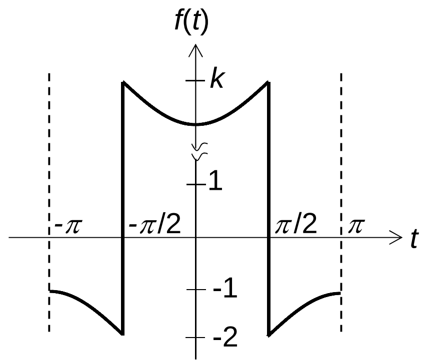
16. The period of a periodic function $f(t)$ is defined as:

$f(t) = \cos(t + \pi) - 2, \quad -\pi < t < -\pi/2$

$f(t) = -\cos(t) + k, \quad -\pi/2 < t < +\pi/2$

$f(t) = \cos(t - \pi) - 2, \quad \pi/2 < t < \pi$

Derive the trigonometric Fourier series expansion of $f(t)$, assuming $k = 3$.



Solution:

$$a_o = \frac{1}{\pi}\left[\int_0^{\pi/2}(-\cos t + k)dt + \int_{\pi/2}^{\pi}\left(\cos(t-\pi) - 2\right)dt\right] =$$

$$\frac{1}{\pi}\left[k\int_0^{\pi/2}dt - 2\int_{\pi/2}^{\pi}dt - \int_0^{\pi/2}\cos t dt - \int_{\pi/2}^{\pi}\cos t dt\right] = \frac{1}{\pi}\left[k\int_0^{\pi/2}dt - 2\int_{\pi/2}^{\pi}dt\right] = \frac{1}{\pi}\left[\frac{k\pi}{2} - \pi\right] = \frac{k}{2} - 1.$$

$$a_n = \frac{2}{\pi}\left[\int_0^{\pi/2}(-\cos t + k)\cos nt dt + \int_{\pi/2}^{\pi}\left(\cos(t-\pi) - 2\right)\cos nt dt\right] =$$

$$\frac{2}{\pi}\left[-\int_0^{\pi}\cos t \cos nt dt + \int_0^{\pi/2}k\cos nt dt - \int_{\pi/2}^{\pi}2\cos nt dt\right] =$$

$$\frac{2}{\pi}\left[-\frac{1}{2}\int_0^{\pi}\cos(n-1)t dt - \frac{1}{2}\int_0^{\pi}\cos(n+1)t dt + k\int_0^{\pi/2}\cos nt dt - 2\int_{\pi/2}^{\pi}\cos nt dt\right] =$$

$$-\frac{1}{\pi}\left[\frac{\sin(n-1)t}{n-1} + \frac{\sin(n+1)t}{n+1}\right]_0^{\pi} + \frac{2k}{n\pi}\left[\sin nt\right]_0^{\pi/2} - \frac{4}{n\pi}\left[\sin nt\right]_{\pi/2}^{\pi} =$$

$0 - 0 + \frac{2k}{n\pi}\sin\frac{n\pi}{2} - 0 - 0 + \frac{4}{n\pi}\sin\frac{n\pi}{2} = \frac{2(k+2)}{n\pi}\sin\frac{n\pi}{2}$. a_n is zero for even values, and the odd harmonics alternate in sign. Thus,

$$f(t)=\left(\frac{k}{2}-1\right)\frac{1}{2}+\frac{2(k+2)}{\pi}\left(\cos t-\frac{1}{3}\cos 3t+\frac{1}{5}\cos 5t-\frac{1}{7}\cos 7t+...\right)$$

Version 1: $k = 3$, $f(t)=\frac{1}{2}+\frac{10}{\pi}\left(\cos t-\frac{1}{3}\cos 3t+\frac{1}{5}\cos 5t-\frac{1}{7}\cos 7t+...\right)$

Version 2: $k = 4$, $f(t)=1+\frac{12}{\pi}\left(\cos t-\frac{1}{3}\cos 3t+\frac{1}{5}\cos 5t-\frac{1}{7}\cos 7t+...\right)$

Version 3: $k = 5$, $f(t)=\frac{3}{2}+\frac{14}{\pi}\left(\cos t-\frac{1}{3}\cos 3t+\frac{1}{5}\cos 5t-\frac{1}{7}\cos 7t+...\right)$

Version 4: $k = 6$, $f(t)=2+\frac{16}{\pi}\left(\cos t-\frac{1}{3}\cos 3t+\frac{1}{5}\cos 5t-\frac{1}{7}\cos 7t+...\right)$

Version 5: $k = 7$, $f(t)=\frac{5}{2}+\frac{18}{\pi}\left(\cos t-\frac{1}{3}\cos 3t+\frac{1}{5}\cos 5t-\frac{1}{7}\cos 7t+...\right)$

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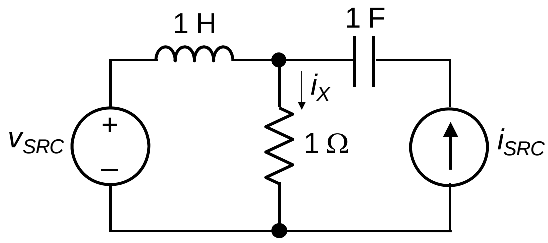
17. Given $v_{SRC} = \cos t$ V and $i_{SRC} = \sin 2t$ A.

6% (a) Derive the expression for i_X in the time domain.

5% (b) Determine the power dissipated in the resistor.



Solution: (a) Let the amplitude of v_{SRC} and i_{SRC} be K. With the current source replaced by an open circuit, $\mathbf{I_{x1}} = \frac{K\angle 0^\circ}{1}\frac{1}{1+j}=\frac{K}{2}(1-j)$; $i_{X1} = \frac{K}{\sqrt{2}}\cos(t-45^\circ)$ A. With the current source replaced by a short circuit, $\mathbf{I_{x2}} = K\angle 0^\circ\frac{j2}{1+j2}=\frac{2K}{\sqrt{5}}\angle(90^\circ-\tan^{-1}2)$; $i_{X2} = \frac{2K}{\sqrt{5}}\sin(2t+26.57^\circ)$ A. Hence, $i_X = \frac{K}{\sqrt{2}}\cos(t-45^\circ) + \frac{2K}{\sqrt{5}}\sin(2t+26.57^\circ)$ A.

(b) Power dissipated is $P = \frac{1}{2}\left(\frac{K^2}{2}+\frac{4K^2}{5}\right)=0.65K^2$ W.

Version 1: $K = 1$, $0.71\cos(t-45^\circ) + 0.89\sin(2t+26.57^\circ)$ A, $P = 0.65$ W

Version 2: $K = 2$, $1.14\cos(t-45^\circ) + 1.79\sin(2t+26.57^\circ)$ A, $P = 2.60$ W

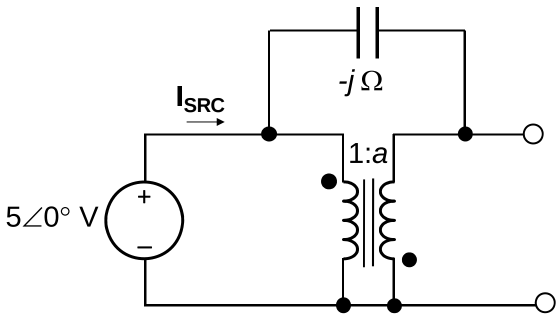
Version 3: $K = 3$, $2.12\cos(t-45^\circ) + 2.68\sin(2t+26.57^\circ)$ A, $P = 5.85$ W

Version 4: $K = 4$, $2.83\cos(t-45^\circ) + 3.58\sin(2t+26.57^\circ)$ A, $P = 10.4$ W

Version 5: $K = 5$, $3.54\cos(t-45^\circ) + 4.47\sin(2t+26.57^\circ)$ A, $P = 16.25$ W

11%

18. Given the circuit shown, with a = 1.



3% (a) Determine the current in the capacitor

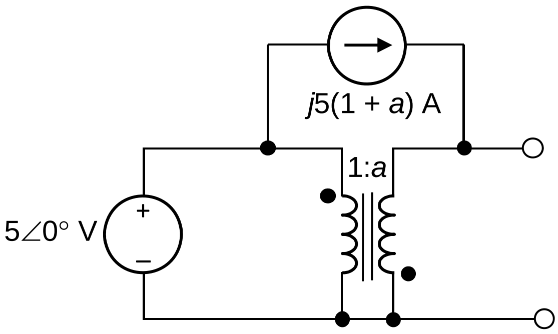
2% (b) Replace the capacitor by a current source, in accordance with the substitution theorem

3% (c) Rearrange the current source as two current sources across the transformer windings

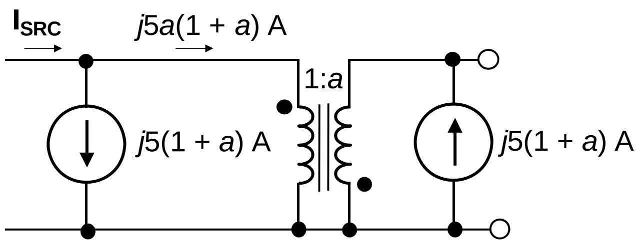
3% (d) Determine $\mathbf{I}_{SRC}$.

Solution: (a) The voltage across the capacitor is 5(1 + a) V. The current through the capacitor is j5(1 + a) A directed from primary to secondary.

(b)



(c)



(d) It follows that $\mathbf{I}_{SRC} = j5(a + 1)^2$ A.

Version 1: a = 2, $\mathbf{I}_{SRC} = j5(2 + 1)^2 = j45$ A

Version 2: a = 3, $\mathbf{I}_{SRC} = j5(3 + 1)^2 = j80$ A

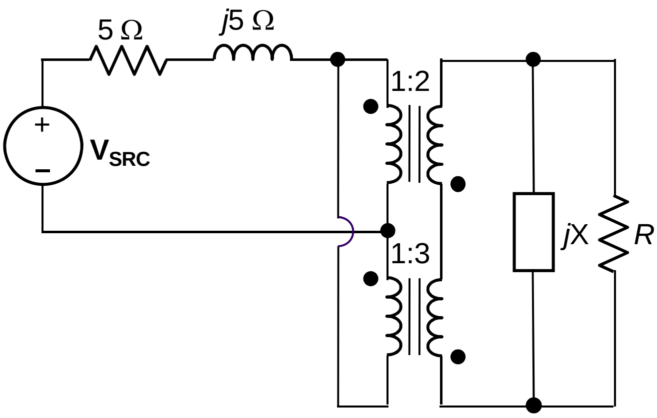
Version 3: a = 4, $\mathbf{I}_{SRC} = j5(4 + 1)^2 = j125$ A

Version 4: a = 5, $\mathbf{I}_{SRC} = j5(5 + 1)^2 = j180$ A

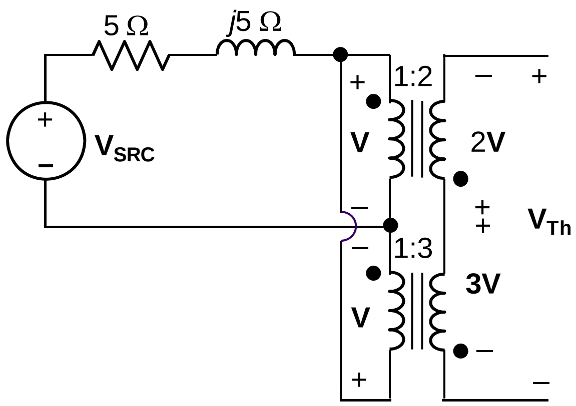
Version 5: a = 6, $\mathbf{I}_{SRC} = j5(6 + 1)^2 = j245$ A

11%

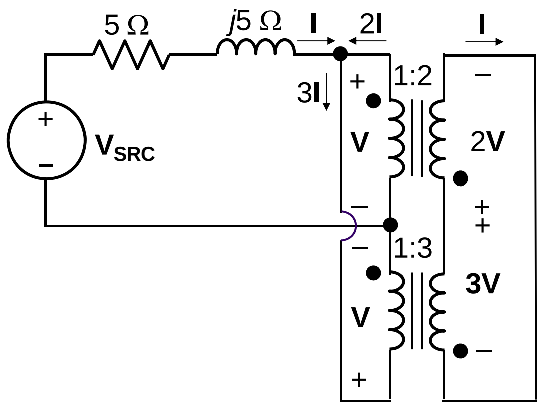
19. Determine X and R for maximum power transfer to R and calculate this power. Assume $\mathbf{V}_{SRC} = 4\angle 0°$ V rms.



Solution: On open circuit, $\mathbf{V_{TH}} = \mathbf{V} = \mathbf{V_{SRC}}$. On short circuit, $\mathbf{V}$ = 0 and $\mathbf{I_N} = \mathbf{I} = \dfrac{\mathbf{V_{SRC}}}{5(1+j)}$. $Y_N = \dfrac{1}{5(1+j)}$ = 0.1(1 – j) S. For maximum power transfer, G_L = 0.1 S, or R = 10 Ω, and B = 0.1 S, or X = -10 Ω.



Under conditions of maximum power transfer, the current in R is $0.5|\mathbf{I_N}| = \dfrac{0.5\,|\mathbf{V_{SRC}}|}{10}$ and the power transferred is $\dfrac{0.25\,|\mathbf{V_{SRC}}|^2}{10} = \dfrac{|\mathbf{V_{SRC}}|^2}{40}$ W.



Version 1: $\mathbf{V_{SRC}} = 4\angle 0^\circ$ rms, $P = \dfrac{16}{40} = 0.4$ W

Version 2: $\mathbf{V_{SRC}} = 5\angle 0^\circ$ rms, $P = \dfrac{25}{40} = 0.625$ W

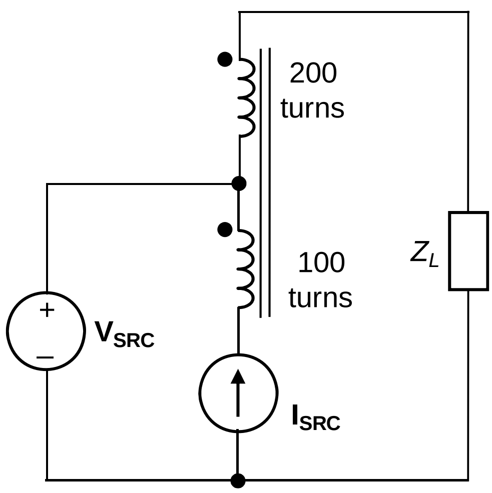
Version 3: $\mathbf{V_{SRC}} = 6\angle 0^\circ$ rms, $P = \dfrac{36}{40} = 0.9$ W

Version 4: $\mathbf{V_{SRC}} = 7\angle 0^\circ$ rms, $P = \dfrac{49}{40} = 1.225$ W

Version 5: $\mathbf{V_{SRC}} = 8\angle 0^\circ$ rms, $P = \dfrac{64}{40} = 1.6$ W.

11%

20. Determine the complex power delivered by each source given that $\mathbf{V_{SRC}} = 5\cos\omega t$, $\mathbf{I_{SRC}} = -2\sin\omega t$, and assuming $Z_L = k(1 - j)$ where k = 1.



Solution: The currents and voltages are as shown. Equating mmfs: $100\times 2\angle 90^\circ + 200\mathbf{I_L} = 0$, or $j2 = -2\mathbf{I_L}$, and $\mathbf{I_L} = -j$ A, $\mathbf{I_1} = \mathbf{I_L} - j2 = -j3$ A.

$\mathbf{V_L} = Z_L\mathbf{I_L} = -jk(1 - j) = -k(1 + j)$ V. $\mathbf{V_2} = \mathbf{V_L} - 5 = -(k + 5) - jk$. $\mathbf{V_1} = \mathbf{V_2}/2 = -(k + 5)/2 - jk/2$ V. $\mathbf{V_I} = 5 - \mathbf{V_1} = (15 + k)/2 + jk/2$ V.

Power delivered by voltage source = $S_v = \dfrac{V_{src}}{\sqrt{2}}\dfrac{I_1^*}{\sqrt{2}} = \dfrac{1}{2}(5)(j3) = \dfrac{j15}{2}$ VA

Power delivered by current source S_I =

$$\frac{V_I}{\sqrt{2}}\frac{I^*_{SRC}}{\sqrt{2}} = \frac{1}{2}\left(\frac{15+k}{2}+\frac{jk}{2}\right)(-j2) =$$

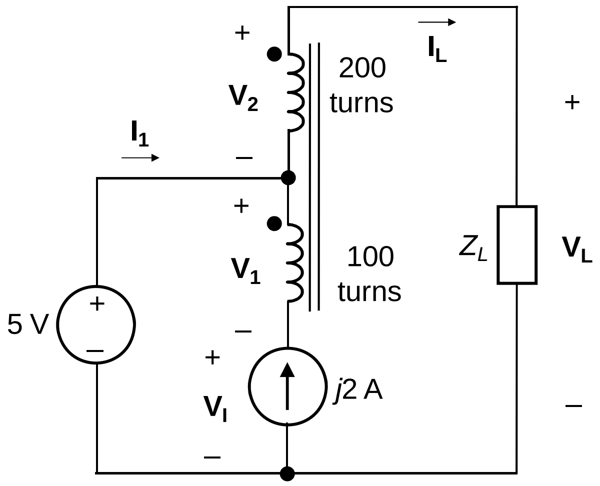
$$\frac{1}{2}\left(k - j(15+k)\right) \text{ VA}$$

Total power delivered by sources =

$$\frac{1}{2}\left(j15 + k - j15 - jk\right) = \frac{k}{2}(1-j)$$

As a check, S_L =

$$\frac{V_{Lm}}{\sqrt{2}}\frac{I^*_{Lm}}{\sqrt{2}} = \frac{1}{2}\left(-jk(1-j)\right)(j) = \frac{k}{2}(1-j) \text{ VA.}$$



Version 1: $k = 1$, $S_v = j7.5$ VA; $S_I = 0.5 - j8$ VA

Version 2: $k = 2$, $S_v = j7.5$ VA; $S_I = 1 - j8.5$ VA

Version 3: $k = 3$, $S_v = j7.5$ VA; $S_I = 1.5 - j9$ VA

Version 4: $k = 4$, $S_v = j7.5$ VA; $S_I = 2 - j9.5$ VA

Version 5: $k = 5$, $S_v = j7.5$ VA; $S_I = 2.5 - j10$ VA